Algorithm for the calculation of power flow for unbalanced distribution grids through the backward/forward sweep method

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Abstract. This paper presents the development of a load flow algorithm for unbalanced radial distribution grid using the backward/forward sweep method, implemented through Matlab® software. The purpose of this methodology is to use the Kirchoff´s laws of voltage and current, as well the Ohm's law, so that the total system losses and the nodal voltages are encountered. This methodology presents satisfactory results regarding characteristic of the load (unbalanced loads) and reduced analysis time (compared to the other published methodology). In the algorithm three load methodologies and simulations with capacitors connected to the lines were implemented: the first one load methodology with 2kVA of load, the second one with the nominal power of the transformer and the third one with 50% of load. After successful algorithm testes with a published unbalanced system with 34 buses, simulations with a real low voltage distribution grid were made. This paper is organized in the following way: it begins with a brief introduction; the methodology used is summarized; the steps of the power flow algorithm are shown; followed by simulations performed on a typical 30 kVA distribution transformer; and finally, the results and conclusions about the study.

Keywords: Load Flow, Backward/Forward Sweep, Radial Distribution Grid.

1 Introduction

The importance of electric energy in society has demanded from the power distributors increasing quality levels in the services provided by them. According to the Empresa de Pesquisa Energética (EPE), electricity consumption in Brazil will increase by 3.7% per year by 2026 [1]. Then, the concessionaries and licensees electric energy are required to plan and/or restructure their distribution grids in order to attend all the consumers within the parameters required to the Angência Nacional de Energia Elétrica (ANEEL) [2].

The planning of the distribution system is a process of study and analysis that a power distributor performs to ensure that their grids are reliable and that the energy supplied to the consumers has the minimum conditions required by ANEEL. This study is essential to ensure that changes in energy demand keep the system operating in a technically and economically feasible way. For this, we need fast and economic planning tools, in order to evaluate the consequences of a change of the rest of the system.

For the correct planning of the distribution grid, it starts with obtaining the characteristics of the loads to be installed, like: demand, utilization factor, type of grid connection. From these, a study is made to verify if the grid to be interconnected the load will be support the increase of demand [4]. However, for many years, few tools were used, so that the system was oversized, providing the necessary loading. However, the consumption is often bellow the nominal power of the distribution transformer [5].

One of the most used ways of adapting new loads to systems are the load flow methods. The most used methods by distribution companies are based in Gauss-Seidel and Newton-Raphson methods. However, these methods were developed to transmission systems, in which approximations are made to facilitate the calculations to be performed [6]. One of the alternatives for more accurate studies in unbalanced radial systems is the use of the Backward/Forward methodology, which is used in radial systems and is characterized to utilization of Kirchhoff's laws and Ohm's law in the steps of its resolution [7].

2 Backward/Forward Methodology

The power flow calculation of an electric system is of paramount importance, since knowing the state of the system it's necessary to solve almost of the problems related to it [8]. The load flow techniques can be divided in two classes: Newton's method and methods based on Ohm's law and Kirchhoff's laws [9].

Since the invention and diffusion of computers, since the 1950s, many load flow resolution methods were developed. However, the vast majority of contributions are for transmission systems only [9]. However, given the characteristics of distribution systems such as:

- Radial structure;
- Unbalanced grid grounded or not;
- Distributed generation;
- Unbalanced loads;
- High numbers of loads and systems buses; and
- Relation between resistance and reactance of raise lines, and so on [10];

the widely used techniques don't converge [11].

In other side, this techniques help in the definition of other methods to be used. In this context, the Backward/Forward sweep methodology is being studied and tested in radials structures [12].

If the system is totally radial, like in Fig. 1, it's known that de current travels only in one direction. The steps for using the Backward/Forward method for the sum of the currents are [7]:

- Starts all nodes with voltage in 1pu;
- Based in the nodal voltages, the load current and line current are calculated;
- The total current flowing through each stretch of lines is calculated. The calculation starts from sending bus (SB) to ending bus (EB) – backward step;
- The new nodal voltages are calculated in each node, starting from SB to EB – forward step;
- If the values found of this difference are smaller than a stipulated maximum error, it ends the iteration. Otherwise, steps 2, 3, 4 and 5 are repeated.

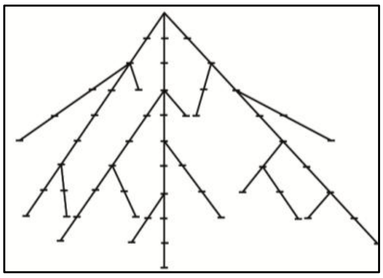

Fig. 1. A typical radial system of electric energy distribution

3 Algorithm Development

The development of the algorithm consists of steps, in which each implementation of new functions and calculations is tested. Below, which step of the process is described separately:

i. Prior to the initialization of the load flow algorithm, it's necessary to input matrices with the characteristics of the system being studied. For the data entry, two input matrices were implemented: BRANCHES, with the conductors data and distance between the nodes and; LOADS, with the loads data presents in the system.

   After the programmer starts the algorithm, it requests four input data in addition to the matrices with the characteristics of the system. They are: the maximum acceptable difference between the voltages of the nodes; the maximum number of iterations, since the system may not converge; soil resistivity in Ω.m and; frequency of the study grid. The first two will be used in the iterative calculations, and the other ones, will be used to determine the values of the impedance matrices.

ii. After the data inputs, the algorithm initializes the computations of the own and mutual impedances. Since in this study the soil resistivity is considered homogeneous in the analyzes and the distance between the phases and neutral is less than 15% of the suppose return by ground, the own and mutual inductances of the phases and neutral are obtained by the Carson Clen equations, show in Eq. (1) and (2) [13].

$$Z_{ii} = R_i + R_{earth} + j2\omega 10^{-4} ln\left(\frac{D_e}{r_i}\right) \left[\frac{\Omega}{km}\right] \quad (1)$$

$$Z_{ij} = R_{earth} + j2\omega 10^{-4} ln\left(\frac{D_e}{d_{ij}}\right) \left[\frac{\Omega}{km}\right] \quad (2)$$

Where:

Z_{ii}[Ω/km]: is the own conductor impedance;
Z_{ij}[Ω/km]: is the mutual impedance between the conductors i and j;
R_i[Ω/km]: is the ohmic resistance of the conductor;
ω[rad]: is the angular frequency;
r_i [m]: is the radius of the conductor used;
R_{earth}[Ω/km]: is the resistance of the suppose return by ground, given by Eq. (3), and
D_e[m]: is the depth of the current by the supposed ground report, given by Eq. (4).

$$R_{earth} = \pi^2 f 10^{-4}, \left[\frac{\Omega}{km}\right] \quad (3)$$

$$D_e = 659\sqrt{\frac{\rho}{f}}, [m] \quad (4)$$

For the 4-wire system, show in Fig. 2, the grid inductance matrices is illustrated by Eq. (5). However, as in this the neutral will be considered grounded in all extension, that means, neutral conductor voltage equal to zero, the complete 4x4 matrix can be transformed into 3x3 through the Kron reduction, shown in Eq. (6) [14].

$$Z_{full} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\ Z_{ca} & Z_{bc} & Z_{cc} & Z_{cn} \\ Z_{na} & Z_{bn} & Z_{nc} & Z_{nn} \end{bmatrix} = \begin{bmatrix} [Z_{ij}] & [Z_{in}] \\ [Z_{nj}] & [Z_{nn}] \end{bmatrix} [\Omega/\text{km}) \quad (5)$$

$$Z_{Kron} = [Z_{ij}] - [Z_{in}] x [Z_{nn}]^{-1} x [Z_{nj}] \ [\Omega/\text{km}] \quad (6)$$

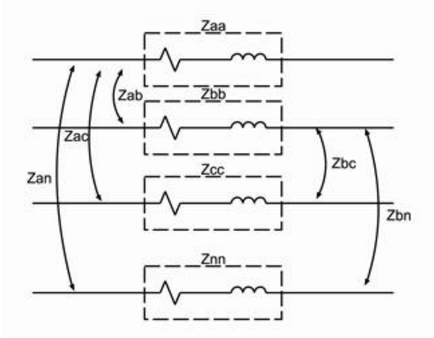

Fig. 2. Own and mutual impedance of a distribution line [15].

iii. For the calculation of the currents at the nodes, loads with constant active and reactive powers were used, i.e., these do not change during the iterations. Thus, as voltage drops occur in the lines, the currents flowing into the load increase. Eq. (7) presents the calculation of the load currents considering constant power [5].

$$\begin{bmatrix} I_{carga_a} \\ I_{carga_b} \\ I_{carga_c} \end{bmatrix} = \begin{bmatrix} \frac{P_a - jQ_a}{V_a^*} \\ \frac{P_b - jQ_b}{V_b^*} \\ \frac{P_c - jQ_c}{V_c^*} \end{bmatrix} \tag{7}$$

Where:

$P_{abc}[W]$: is the active power of the load in phase A, B or C;

Q_{abc}[VAr]: is the reactive power of the load in phase A, B or C; and;

$V_{abc}^*[V]$: is the voltage coupled in phase A, B or C.

iv. In distribution systems, more than one branch can derive from a bus [5].Thus, before beginning the calculations of the node currents, it becomes necessary to know the connectivity of the systems buses. For this propose, the methodology described in [8] was used which 2 vectors mf [i] and mt [i] were implemented. The vector mf [i] store the initial value for de bus and the vector mt [i] memorizes how many links this bus has. Each node can be divided into the final node, intermediate node or join node by the equations:

- $mt[i] - mf[i] = 0$: final node. With the exception of the source node;
- $mt[i] - mf[i] = 1$: intermediate node and;
- $mt[i] - mf[i] > 1$: juction node.

With the knowledge of the connections of the systems and knowing all the load currents, the currents that flow in the buses can be found by means of Eq. (8).

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_{ij}^{n} = \begin{bmatrix} I_{carga_a} \\ I_{carga_b} \\ I_{carga_c} \end{bmatrix}_{j}^{n} + \sum_{k=j+1}^{if} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_{jk}^{n} \text{ [A]} \tag{8}$$

Where:

$[I_{abc}]_{ij}$ [A]: are the currents in the phases A, B and C in the branch comprising nodes i and j;

$[I_{carga_{abc}}]_j$ [A]: are the currents in the phases A, B and C that flow in the j node; and

$[I_{abc}]_{jk}$ [A]: are the currents in the phases A, B and C in the branches subsequent to node j;

if: is the final node of the system;

n: is the number of iterations.

v. Subsequently, the values of the new nodal voltages, obtained by means of Eq. (9), are found.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_{j}^{n} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_{i}^{n} - \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}_{ij} x \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_{ij}^{n} \text{ [V]} \tag{9}$$

Where:

$[V_{abc}]_j$ [V]: are the voltages in the phases A, B and C in the node j;

$[V_{abc}]_i$ [V]: are the voltages in the phases A, B and C in the node i; and

$[Z_{abc}]_{ij}$ [Ω]: is the symmetric matrice of line impedances that connect the buses i and j.

vi. The new voltages are subtracted from the old ones, and the difference between the values is compared to the maximum allowable difference. If the values are greater, repeat iii, iv, v and vi. If not, end the iterations.

vii. The system losses for each branch are calculated through the vector product of the impedances and the current matrice, and the result is multiplied by the current conjugate given by Eq. (10).

$$\begin{bmatrix} S_{loss_a} \\ S_{loss_b} \\ S_{loss_c} \end{bmatrix}_{ij} = \left(\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}_{ij} x \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_{ij} \right) . \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_{ij}^{*} \text{ [VA]} \tag{10}$$

Where:

$\left[S_{loss_{abc}}\right]_{ij}$ [VA]: is the apparent power loss matrice of phases A, B and C between nodes i and j; and

$[I_{abc}]_{ij}^{*}$[A]: is the currents conjugate of phases A, B and C between nodes i and j.

After de development, the circuit presented in [16] was used for the comparison of values and validation of the algorithm. The circuit has only aerial lines with 34 buses, nominal voltage of 24.9 kV, phase transposition, single and three phases and three phase capacitors connected to two buses, as shown in Fig. 3.

The system converges after 7 iterations. Table 1 the data with the greatest difference between the values of the simulation and those presented in [16]. The results are presented with only 3 decimal places in order that the comparison between the values occurs correctly, since in [16] the same number of digits is used.

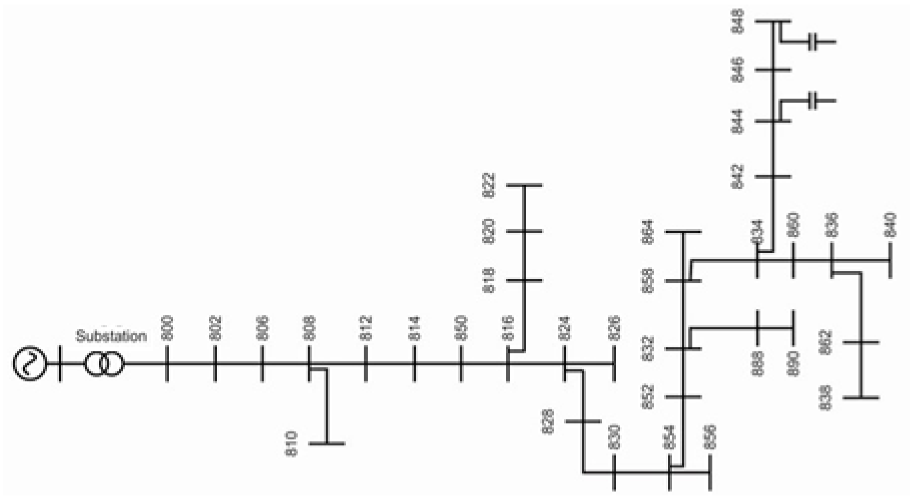

Fig. 3. Modified IEEE-34 system [16].

Table 1. Maximum differences found for the modified IEEE-34 system [From author, 2018].

	p.u	**Absolutes**
Va	0,001	24,9
Θa	0,115	0,115
Vb	0,001	24,9
Θv	0,172	0,172
Vc	0,001	24,9
Θc	0,221	0,221

4 Analysis of Results

The simulated circuit is formed by a typical 30 kVA rural distribution transformer with 18 buses. The system is three-phase and has 6 consumers: 4 single-phase and 2 three-phase. The conductor that makes up the grid is formed of aluminum. The consumption was considered unbalanced for all analyzes, so that the bus connected to the transformer presented different power deliveries for each phase.

Since it is a distribution transformer, the voltage of 380/220 V will be used as the initial in all buses. Fig. 4 shows the unifilar of the simulated system.

For the simulation data, an error equal to 0.0001 V and maximum number of 100 iterations was used as the criterion for stopping. The soil resistivity used was 100 Ω.m.

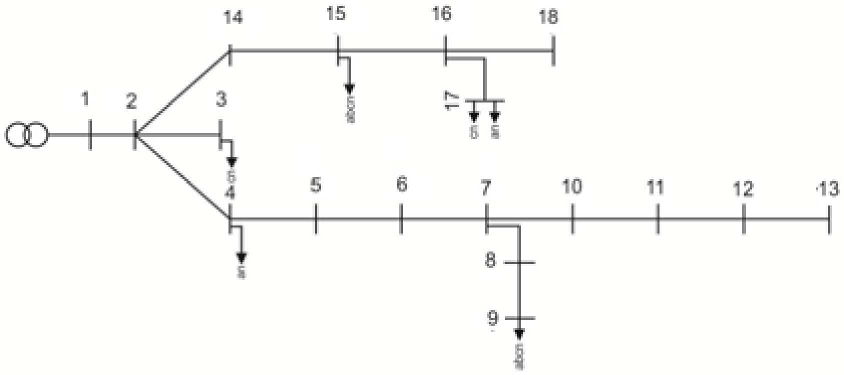

Fig. 4. Modified IEEE-34 system [16].

In this work, three loading hypotheses were analyzed: one with loading bellow 2 kVA; a second with nominal power in the transformer; and finally a hypothesis with 50% loading. The loads will be distributed randomly among consumers. In each analysis the loads will be treated in different ways and the power factors will be different between the hypotheses.

For the first analyses, higher consumption was used in the consumer connected to the system bus 15, so that among the three phases, phase A was considered to be the higher load. The total system load was 1.9 kW for the active power, that means, low power consumption. In this analysis, the loads were considered as constant and single-phase power.

After simulation using MATLAB®, it is noted that in theoretical values, the maximum voltage drop is 1.47, that is, drop of 0.67% of the nominal voltage. Due the mutual inductance between the lines and the phase lag of 120º between phases, in low-flow branches there is a voltage rise between the buses, rather than a drop. This fact stems from the mutual inductance generated by the other conductors.

The system converges after 4 iterations. The energy supplied by bus 1 was 1.91 kW for the simulation and the losses were at 18.52 kW. Since in phase C the charge is high compared to phase B, the currents flowing from the transformer in phases A and C were greater than 90% of the total. Fig. 5 shows the current of each branch after convergence. The values are in p.u., whose base was the sum of the currents in the 3 phases downstream of bus 1.

Because bus 1 has constant and symmetrical voltage, the voltage unbalanced in this bus is zero. For the simulation, the imbalance did not exceed 0.9%, as can be observed in Fig. 6. It is noted that the voltage unbalance increases the further the bus is from the transformer.

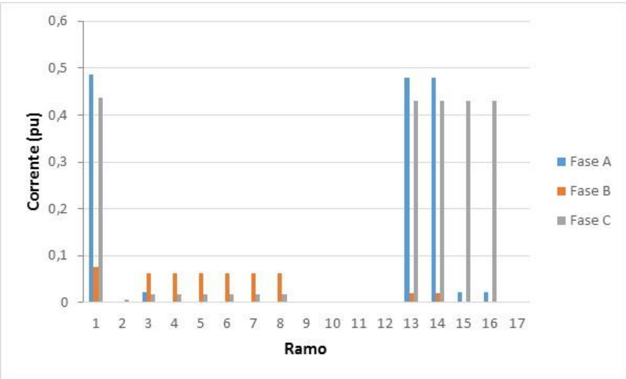

Fig. 5. Currents at each node after convergence for the first simulation [From author, 2018].

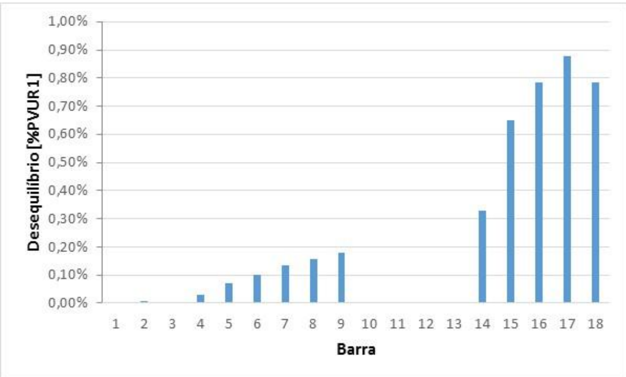

Fig. 6. Unbalance of voltage after the first simulation [From author, 2018].

For the calculation of the voltage unbalance shown in Fig. 6, we used Eq. (11), which denotes the maximum deviation between the highest and the lowest system voltage divided by the mean in the three phases. The methodology uses only effective voltage for the calculation. The unbalance was calculated only in the buses where the 3 phases of the system are found.

$$\%PVUR1 = \frac{máx(V_a, V_b, V_c) - mín(V_a, V_b, V_c)}{V_m} \tag{11}$$

$$V_m = \frac{V_a + V_b + V_c}{3} \tag{12}$$

Where:

$\%PVUR1$ [%]: is the voltage unbalance according to the IEEE-936-1987 standard;

$máx(V_a, V_b, V_c)$ [V]: is the maximum value between the voltages A, B and C;

$mín(V_a, V_b, V_c)$ [V]: is the minimum value between the voltages A, B and C; and

V_m [V]: is the mean of the voltage in three phases, given by Eq. (12).

For the second analysis, 30 kVA was used as the sum of the powers delivered in the loads. In this simulation phase B had the highest load, with a value of 12 kVA. Phases A and C obtained potencies of 9 kVA each. The loads were distributed randomly among the consumers and the loads were considered as constant impedance.

The system converges after 6 iterations, where de maximum voltage drop was approximately 11V in phase B of bus 9. This value was the highest found in all simulations. However, this is within the appropriate voltage values established by ANEEL [18].

In this simulation, high losses were observed in comparison to the first analysis, with values of 675 W, which means, 2.2% of the total energy supplied by bus 1. About the imbalance, it is noticed that due to the voltage drops in the 3 phases due to the unbalanced currents (Fig. 7), values were close to 5% for Eq. (11).

Since loads of constant impedance were used, the values of the powers delivered to the loads did not present equal values to the initial ones. The largest difference was obtained in bus 9 of the system, in which the difference was established close to 700VA, which means, 10% of the pre-established total. This fact reflects the greatest voltage drop in this bus. In the other nodes of the system, the differences did not exceed 4%.

For a last analysis, loads with a power factor of 0.8 and constant current was simulated. In this hypothesis, the highest loading phase was phase A, with 8kVA followed by phases C and B with loads of 4kVA and 3kVA, respectively. The buses with the highest delivery power are buses 3 and 4 of the system.

After the convergence of values, it is noticed that the active losses of the system presented values of 128W. In percentages, the losses are low in relation to the second analysis. This fact is due to the way of the currents, since in the previous analysis the way was greater due to the presence of loads away from the transformer. In the last analysis, the higher power loads are close to the first bus. Thus the currents of greater magnitude traveled only the initial nodes of the system, as shown in Fig. 8.

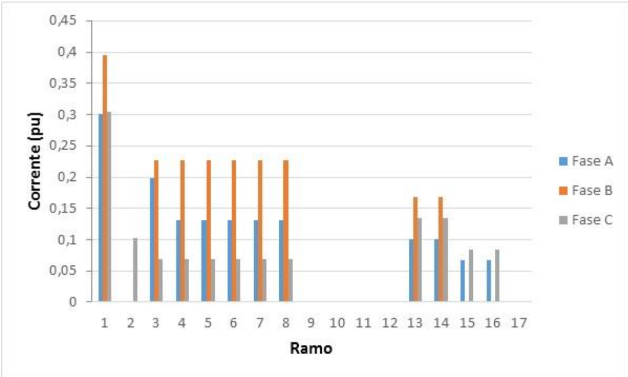

Fig. 7. Currents at each node after convergence to the second simulation [From author, 2018].

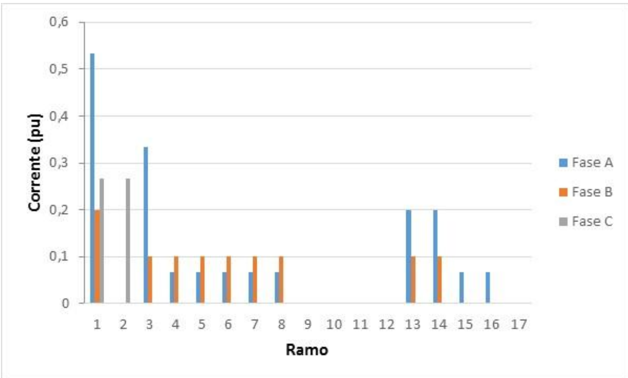

Fig. 8. Currents at each node after convergence to the third simulation [From author, 2018].

The maximum voltage drop obtained in this simulation was 1.7%. This value was observed in phase A of bus 9 of the system. In the other phases, the fall did not exceed 0.91%. For the voltage unbalance, it is observed that the maximum variation between the three phases also occurs at node 9: 2.05%.

Analogously to the second simulation, the power delivered to the loads differs from the input matrices. After convergence of values, the greatest difference in absolute values between the initial and found powers was approximately 21VA at bus 3. However, in percentage values, the biggest difference occurred in bus 9. However, it is observed that for this modeling of load on current module constants, the differences between the powers of initial loads and after convergence are smaller in relation to constant impedance loads used in the second simulation.

5 Conclusions

From the results obtained in this work, it is observed that the developed algorithm shows robustness in the methodology used since it allows using unbalanced loads and connected capacitors phase-phase and phase-neutral.

When comparing the algorithm developed with unbalanced model of 34 buses published, convergence was obtained in only 7 iterations. The values obtained were close to those presented, differing only in the last significant algorism. Thus, the algorithm is validated in this work presented.

In the simulations carried out, although it contains load unbalanced in all the configurations, it is observed that the algorithm can be applied to the analyzed system successfully. The voltage unbalanced and the losses were higher in the analysis where the higher transformer load was simulated.

The difference obtain by comparing data found via software to the field can indicate points in the circuit where losses are remediable, increasing the efficiency and supply of the transformer. The simulation of the distribution circuits then proves to be a valuable tool for the analysis and minimization of technical and non-technical losses.

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